THE SHOCK-WAVE STRUCTURE FOR ARBITRARY PRANDTL NUMBERS AND HIGH MACH NUMBERS

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Abstract. By use of a modified law of viscosity of the form $\mu = T^n$, where n is a given function of M, an analytical solution is obtained to the problem of strong shock waves. These analytical results agree very well with results obtained by using Boltzmann equation to the same problem.

1. Introduction

The normal shock-wave structure may be described by one Cartesian coordinate x extending from minus infinity to plus infinity. In our following treatment nothing is considered about the order of the thickness of the region whether it is small or not. This gives us the right to use the Navier–Stokes equations.

In 1922, Becker (1922) obtained an exact solution of a system of one-dimensional equations of a real fluid with Prandtl number $\frac{3}{4}$, by assuming that the enthalpy is a function of the velocity in Equation (15). Thomas (1944) subsequently extended Becker's results to variable coefficients of viscosity and heat conduction.

In the present work, the following modifications are made:

(1) The enthalpy-velocity relationship is suggested in the form (17).

(2) The velocity at the inflection point is taken in the form (22).

(3) The viscosity law is given in Equations (23).

The above modifications enable us to arrive good agreement for the thickness of the shock-wave obtained from Navier–Stokes theory and that obtained from the kinetic theory.

For the distributions of the variables inside the transition region, Equations (24)–(27) for velocity, density, enthalpy, and pressure are proposed.

2. Basic Equations

The steady, one-dimensional flow, parallel to x-axis, of viscous, heat-conducting, compressible fluid is described by the following hydrodynamic equations in dimensional form (cf. Pai, 1959)

$$\frac{\mathrm{d}}{\mathrm{d}x'}(\rho' u') = 0\,, \tag{1}$$

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